Problem of the Week
Problem B and Solution
Traffic Predictions

Problem

Petr was standing at the bus stop during rush hour and started counting the passing vehicles. In the first five minutes he waited, he counted 20 cars, 25 vans and 15 trucks.

(a) Based on Petr's sample data, what is the theoretical probability that the next vehicle will be a truck?

(b) Petr counted vehicles for another five minutes and discovered that the experimental probability of a vehicle being a car was the same for his first and second samples. If Petr counted a total of 84 vehicles in his second sample, how many of those vehicles were cars?

Solution

(a) Petr's sample had a total of $20 + 25 + 15 = 60$ vehicles. Since 15 of these were trucks, the theoretical probability that the next vehicle will be a truck is $\frac{15}{60} = \frac{1}{4}$. Notice that the probability is equal to the fraction of trucks in the sample.

(b) Petr's first sample included 20 cars which is $\frac{20}{60} = \frac{1}{3}$ of the vehicles. Thus, the experimental probability of a vehicle in the first sample being a car is $\frac{1}{3}$. If this experimental probability is the same for the second sample, then $\frac{1}{3}$ of the cars in the second sample must have been cars. Since his second sample had a total of 84 vehicles, and $\frac{1}{3}$ of 84 is $\frac{1}{3} \times 84 = 28$, it follows that 28 of the vehicles in the second sample were cars.

NOTE: You cannot predict the individual numbers of vans or trucks in the second sample, because you don't know the experimental probabilities of a vehicle being a van or a truck for the second sample.

EXTENSION: If Petr had determined that the probability of a vehicle being a car was the same for his first and second samples, would it have been possible for him to have observed 85 vehicles in the second sample?