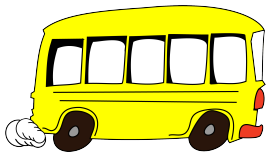

Problem of the Week
Problem C and Solution
Teacher Road Trip 1

Problem

To help pass time on a long bus ride, 35 math teachers created a sequence of numbers, with each teacher saying one term in the sequence. The first teacher said the number 2, the second teacher said the number 8, and every teacher after that said the sum of the two previous terms. Thus,

- the third teacher said the sum of the first and second terms, which is $2 + 8 = 10$, and
- the fourth teacher said the sum of the second and third terms, which is $8 + 10 = 18$.

Once the final teacher said their number, the 25^{th} teacher announced they had made a mistake and their number should have been one more than what they had said. How much larger should the final teacher's number have been?

Solution

Solution 1

We will write out the sequence of numbers the teachers actually said, and then the sequence of numbers they should have said, and then find the difference between the last term in each sequence.

Here are the first 24 numbers that the teachers said:

2, 8, 10, 18, 28, 46, 74, 120, 194, 314, 508, 822, 1330, 2152, 3482, 5634, 9116, 14 750, 23 866, 38 616, 62 482, 101 098, 163 580, 264 678

Here are the correct 25^{th} to 35^{th} numbers that the teachers should have said:

428 258, 692 936, 1 121 194, 1 814 130, 2 935 324, 4 749 454, 7 684 778, 12 434 232, 20 119 010, 32 553 242, 52 672 252

Here are the 25^{th} to 35^{th} numbers that the teachers actually said:

428 257, 692 935, 1 121 192, 1 814 127, 2 935 319, 4 749 446, 7 684 765, 12 434 211, 20 118 976, 32 553 187, 52 672 163

The difference between the correct and incorrect 35^{th} number is $52\,672\,252 - 52\,672\,163 = 89$. Therefore, the 35^{th} number was off by 89, and so the final teacher's number should have been 89 larger than the number they had said.

Solution 2

In this solution we will solve the problem without actually calculating all the terms in the sequence.

We know the 25^{th} term is off by 1. Therefore, the next terms will be as follows.

- The 26^{th} term will also be off by 1 since it equals the sum of the 24^{th} term (which is unchanged) and the 25^{th} term (which is off by 1).
- The 27^{th} term will be off by 2 since it is the sum of the 25^{th} term (which is off by 1) and the 26^{th} term (which is off by 1).
- The 28^{th} term will be off by 3 since it is the sum of the 26^{th} term (which is off by 1) and the 27^{th} term (which is off by 2).

This pattern will continue on, so we can summarize it in a table.

Term Number	Amount Below the Correct Value
24	0
25	1
26	1
27	2
28	3
29	5
30	8
31	13
32	21
33	34
34	55
35	89

Therefore, the 35^{th} term was off by 89, and so the final teacher's number should have been 89 larger than the number they had said.

Notice that the terms in the right column of the table follow the same rule as the original question. That is, each term is the sum of the previous two terms.

For Further Thought: The last 11 numbers in the right column of the table are the first 11 numbers of a famous sequence known as the Fibonacci Sequence. You may wish to investigate the Fibonacci Sequence further.