Problem of the Week
Problem C and Solution
Coin Collection

Problem

Arya has never travelled to another country, but has a collection of foreign coins given to him by friends and family who have. In his collection he has 10 coins from Africa, 6 coins from Asia, 7 coins from South America, and 8 coins from Europe.

One day Arya's grandfather added some Australian coins to the collection. After he did that, he told Arya that if he took a coin at random from the collection, the probability of it being from either Africa or Asia was $\frac{4}{9}$.

How many Australian coins did Arya's grandfather add to the collection?

Solution

In order to determine the probability of a randomly selected coin being from either Africa or Asia, we divide the number of coins from Africa or Asia by the total number of coins in the collection. In other words,

$$\text{Probability of selecting a coin from Africa or Asia} = \frac{\text{Number of coins from Africa or Asia}}{\text{Total number of coins}}$$

From here we will present two different solutions to this problem.

Solution 1

When Arya's grandfather adds Australian coins to the collection, this does not change the number of coins from Africa or Asia. Therefore the number of coins from Africa or Asia in the collection is $10 + 6 = 16$. We are also told that the probability of drawing a coin from Africa or Asia is $\frac{4}{9}$. We can substitute these values into our equation.

$$\text{Probability of selecting a coin from Africa or Asia} = \frac{\text{Number of coins from Africa or Asia}}{\text{Total number of coins}}$$
$$\frac{4}{9} = \frac{16}{\text{Total number of coins}}$$

Since $\frac{4}{9} = \frac{16}{36}$, it follows that

$$\frac{16}{36} = \frac{16}{\text{Total number of coins}}$$

Therefore, the total number of coins in the collection is 36.

Originally there were $10 + 6 + 7 + 8 = 31$ coins in the collection, and then Arya's grandfather added some Australian coins. Since there were 36 coins in the collection after the Australian coins were added, it follows that Arya's grandfather must have added $36 - 31 = 5$ Australian coins to the collection.

Solution 2

This solution uses algebra, which may be beyond what some students at this level are familiar with.

Let n represent the number of Australian coins that Arya's grandfather added to the collection. Then the number of coins from Africa or Asia in the collection is $10 + 6 = 16$, and the total number of coins in the collection is $10 + 6 + 7 + 8 + n = 31 + n$. We are also told that the probability of drawing a coin from Africa or Asia is $\frac{4}{9}$. We can substitute these values into our equation.

$$\text{Probability of selecting a coin from Africa or Asia} = \frac{\text{Number of coins from Africa or Asia}}{\text{Total number of coins}}$$
$$\frac{4}{9} = \frac{16}{31 + n}$$

Since $4 \times 4 = 16$, it follows that $9 \times 4 = 31 + n$. We can simplify and solve this equation to find the value of n.

$$9 \times 4 = 31 + n$$
$$36 = 31 + n$$
$$36 - 31 = n$$
$$5 = n$$

Therefore, Arya's grandfather added 5 Australian coins to the collection.