Problem of the Week
Problem C
Fair Game?

For a school mathematics project, Zesiro and Magomu created a game that uses two special decks of six cards each. The cards in one deck are labelled with the even numbers 2, 4, 6, 8, 10, and 12, and the cards in the other deck are labelled with the odd numbers 1, 3, 5, 7, 9, and 11.

A turn consists of Zesiro randomly choosing a card from the deck with even-numbered labels and Magumo randomly choosing a card from the deck with odd-numbered labels. These two cards make a pair of cards. After a pair of cards is chosen, they perform the following steps.

1. They determine the sum, S, of the numbers on the cards. For example, if Zesiro chooses the card labelled with a 6 and Magumo chooses the card labelled with a 3, then $S = 6 + 3 = 9$.

2. Using S, they determine, D, the digit sum. If S is a single digit number, then D is equal to S. If S is a two-digit number, then D is the sum of the two digits of S. For example, if Zesiro chooses the card labelled with a 6 and Magumo chooses the card labelled with a 3, then the sum and the digit sum are both 9. If Zesiro chooses the card labelled with a 10 and Magumo chooses the card labelled with a 5, then the sum is $S = 10 + 5 = 15$ and the digit sum is $D = 1 + 5 = 6$. If Zesiro chooses the card labelled with a 10 and Magumo chooses the card labelled with a 9, then the sum is $S = 10 + 9 = 19$ and the digit sum is $D = 1 + 9 = 10$.

Zesiro gets a point if the digit sum, D, is a multiple of 4.

Magomu gets a point if the number on one of the cards is a multiple of the number on the other card.

Is this game fair? That is, do Zesiro and Magomu have the same probability of getting a point on any turn? Justify your answer.