Problem of the Week
## Problem C and Solution
The Missing Pieces

Problem

The following information is known about $\triangle PQR$.

- The point S is on side PR and the point T is on side PQ.

- The distance from P to S is equal to the distance from T to Q.

- The distance from S to R is equal to the distance from P to T.

- $\angle PRQ = 40°$ and $\angle PTS = 20°$.

Determine the value of each of the five other interior angles. That is, determine the values of $\angle RPQ$, $\angle STQ$, $\angle TQR$, $\angle RST$, and $\angle PST$.

Solution

First, we let $\angle RPQ$ measure $a°$, $\angle STQ$ measure $b°$, $\angle TQR$ measure $c°$, $\angle RST$ measure $d°$, and $\angle PST$ measure $e°$.
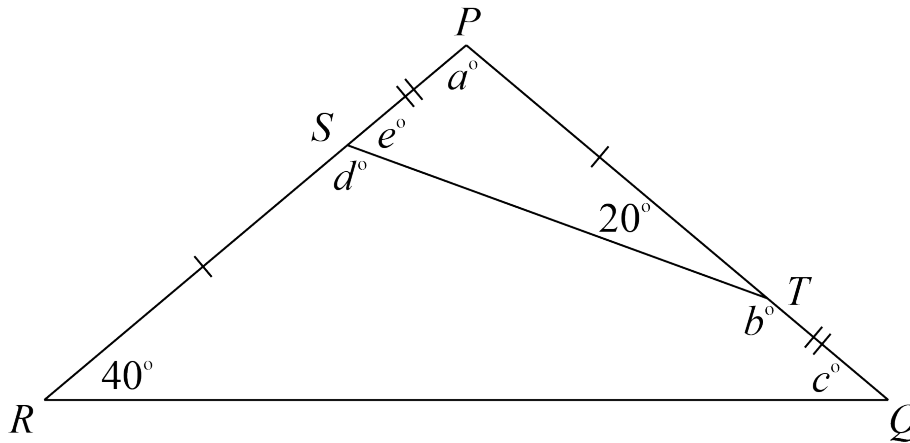
Since $\angle PTQ$ is a straight angle, $20 + b = 180$, and so $b = 160$.

Since $PS = TQ$ and $SR = PT$, it follows that $PS + PR = PT + TQ$, and so $PR = PQ$ and $\triangle PQR$ is isosceles. Therefore $\angle PRQ = \angle PQR$, and so $c = 40$.

Since the angles in a triangle sum to $180°$, in $\triangle PQR$,

$$a + 40 + c = 180$$
$$a + 40 + 40 = 180$$
$$a + 80 = 180$$
$$a = 100$$

Similarly, in $\triangle PST$,

$$a + e + 20 = 180$$
$$100 + e + 20 = 180$$
$$120 + e = 180$$
$$e = 60$$

Since $\angle PSR$ is a straight angle,

$$e + d = 180$$
$$60 + d = 180$$
$$d = 120$$

We have determined the value of all the other five interior angles.
$\angle RPQ = a° = 100°$, $\angle STQ = b° = 160°$, $\angle TQR = c° = 40°$,
$\angle RST = d° = 120°$, and $\angle PST = e° = 60°$.