



Problem of the Week

Problem C and Solution

Just Outside

Problem

In square $OABC$, points A and C lie on the circumference of a circle with centre O , and B lies outside of the circle. Square $OABC$ has an area of 36 m^2 .

Determine the area of the shaded region inside square $OABC$ and outside the circle with centre O , rounded to two decimal places.

Solution

Since $OABC$ is a square with an area of 36 m^2 , its side length must be 6 m . That is, $OA = OC = 6 \text{ m}$.

Since A lies on the circumference of the circle with centre O , the radius of the circle is $r = OA = 6 \text{ m}$.

Therefore, the area of the circle is $\pi \times r^2 = \pi \times 6^2 = 36\pi \text{ m}^2$.

Since $OABC$ is a square, $\angle AOC = 90^\circ$.

Therefore, the area of sector OAC is $\frac{90^\circ}{360^\circ} = \frac{1}{4}$ of the area of the circle.

In other words, the area of the sector OAC is $\frac{1}{4} \times 36\pi = 9\pi \text{ m}^2$.

Therefore,

$$\begin{aligned} \text{Area of shaded region} &= \text{Area of square } OABC - \text{Area of sector } OAC \\ &= 36 - 9\pi \\ &\approx 7.73 \text{ m}^2 \end{aligned}$$

NOTE: In the problem you were asked to give your answer rounded to two decimal places. However, many times in mathematics we are actually interested in the *exact* answer. In this case, the exact answer would be $(36 - 9\pi) \text{ m}^2$.