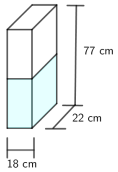

Problem of the Week
Problem C and Solution
Ice Box

Problem

A metal box in the form of a rectangular prism has an 18 cm by 22 cm base and a height of 77 cm. The box is to be filled with water, which will then be frozen. When water freezes it expands by approximately 10%. Determine the maximum depth to which the box can be filled with water so that when the water freezes, the ice does not go above the top of the container.

Solution

Solution 1

To determine the volume of a rectangular prism, we multiply its length, width, and height together. So, the maximum volume of the metal box is

$$18 \times 22 \times 77 = 30\,492 \text{ cm}^3$$

Let the original depth of water in the metal box be h cm.

The water volume before freezing is $18 \times 22 \times h = (396 \times h)$ cm^3. After the water freezes, the volume increases by 10% to 110% of its current volume. That is, after freezing the volume is

$$110\% \text{ of } 396 \times h = 1.1 \times 396 \times h = (435.6 \times h) \text{ cm}^3$$

But the volume after freezing is the maximum volume, 30 492 cm^3. Therefore, $435.6 \times h = 30\,492$ and it follows that $h = 30\,492 \div 435.6 = 70$ cm.

Therefore, the maximum depth to which the box can be filled is 70 cm.

Solution 2

In this solution we note that the length and width remain the same in the volume calculations before and after the water freezes. We need only concern ourselves with the change in the depth of the water.

Let the original depth of water in the container be h cm.

After freezing, the depth increases by 10% to 110% of its depth before freezing. So, after freezing the depth will be 110% of $h = 1.1 \times h = 77$ cm, the maximum height of the container. Then $h = 77 \div 1.1 = 70$ cm.

Therefore, the maximum depth to which the box can be filled is 70 cm.