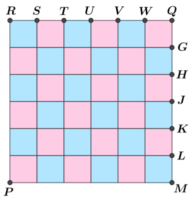


Problem of the Week

Problem C and Solution

All Equal

Problem

Using two cuts, we want to divide the 6 m by 6 m grid shown into three regions of equal area.

One way to do so is by making a horizontal cut through H and a second horizontal cut through K. This method of cutting the grid works, but is not very creative.

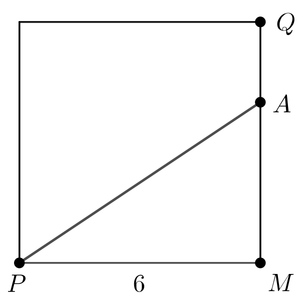
To make things a little more interesting, we must still make two straight cuts, but each cut must start at point P. Each of these two cuts will pass through a point on the outer perimeter of the grid.

Find the length of each cut. Round your answer to one decimal.

Solution

The area of the entire 6 m by 6 m square grid is $6 \times 6 = 36 \text{ m}^2$. Since the square is divided into three regions of equal area, the area of each region must be $\frac{36}{3} = 12 \text{ m}^2$.

Consider the line through P that passes through some point on side QM. Let A be the point where this line intersects QM.



Since $\angle PMQ = 90^\circ$, $\triangle PMA$ is a right-angled triangle with base $PM = 6$ m and height MA. Using the formula area $= \frac{\text{base} \times \text{height}}{2}$, we have area of $\triangle PMA = \frac{6 \times MA}{2} = 3 \times MA$.

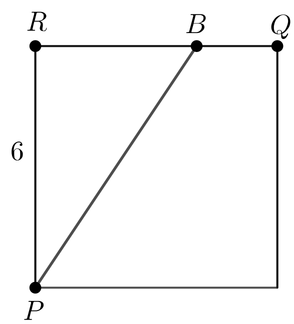
We need the area of $\triangle PMA$ to be 12 m^2. Therefore, $3 \times MA = 12$, and so $MA = 4$ m. Since H is the point on QM with $MH = 4$ m, we must have $A = H$. Therefore, one line passes through the point H.

Since $\triangle PMA$ is a right-angled triangle, using the Pythagorean Theorem we have

$$\begin{aligned} PA^2 &= PM^2 + MA^2 \\ &= 6^2 + 4^2 \\ &= 36 + 16 \\ &= 52 \end{aligned}$$

Therefore, $PA = \sqrt{52} \approx 7.2$, since $PA > 0$.

Consider the line through P that passes through some point on side RQ. Let B be the point where this line intersects RQ.



Since $\angle PRQ = 90^\circ$, $\triangle PRB$ is a right-angled triangle with height $PR = 6$ m and base RB.

Using the formula area $= \dfrac{\text{base} \times \text{height}}{2}$, we have area of $\triangle PRB = \dfrac{RB \times 6}{2} = 3 \times RB$.

We need the area of $\triangle PRB$ to be 12 m^2. Therefore, $3 \times RB = 12$, and so $RB = 4$ m. Since V is the point on RQ with $RV = 4$ m, we must have $B = V$. Therefore, the other line passes through the point V.

Therefore, one line passes through point H and the other passes through point V.

Since $\triangle PRB$ is a right-angled triangle, using the Pythagorean Theorem we have

$$\begin{aligned} PB^2 &= PR^2 + RB^2 \\ &= 6^2 + 4^2 \\ &= 36 + 16 \\ &= 52 \end{aligned}$$

Therefore, $PB = \sqrt{52} \approx 7.2$, since $PB > 0$.

Therefore, the length of each cut is approximately 7.2 m.

Extension:

Try dividing the grid into three regions of equal area using three cuts. (Each cut does not necessarily need to be to the outer perimeter of the grid.)