



Problem of the Week

Problem C and Solution

Divisors and Number

Problem

Your friend Cael always likes challenging you. One challenge is called “*Divisors and Number*”. Cael will tell you certain facts about the divisors of a number and then challenge you to find the number. Here is Cael’s challenge.

“I am looking for a positive integer with exactly eight positive divisors, two of which are 21 and 33.”

Determine Cael’s number.

Solution

Let n represent the number we are looking for.

We know that four of the positive divisors of n are 1, 21, 33, and n . In our solution we will first find the remaining four positive divisors and then determine n .

Since 21 is a divisor of n and $21 = 3 \times 7$, then 3 and 7 must also be divisors of n .

Since 33 is a divisor of n and $33 = 3 \times 11$, then 11 must also be a divisor of n .

Since 7 is a divisor of n and 11 is a divisor of n , and since 7 and 11 have no common divisors, then $7 \times 11 = 77$ must also be a divisor of n .

We have found all eight of the positive divisors of the unknown number. The positive divisors are 1, 3, 7, 11, 21, 33, 77, and n . We now need to determine n .

From the list of divisors, we can see that the prime factors of n are 3, 7, and 11. It follows that $n = 3 \times 7 \times 11 = 231$.

Therefore, Cael’s number is 231.