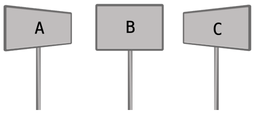

Problem of the Week

Problem C and Solution

Go Back

Problem

Gwen has been given the ability to time travel by walking along three different trails. She can walk on any trail as often as she wishes, but can only walk on one trail at a time. She must walk on the trails using the following rules.

- When she walks on Trail A, she must take 7 steps forward. This will allow her to travel 4 months backward in time.
- When she walks on Trail B, she must take 5 steps backward. This will allow her to travel 7 months backward in time.
- When she walks on Trail C, she must take 2 steps forward. This will allow her to travel 3 months backward in time.

One day she travels 5 years into the past. She made a total of 25 steps backward and walked on the three trails a total of 12 times. How many steps forward did she take?

Solution

Gwen travelled 5 years back in time, which is equivalent to travelling $5 \times 12 = 60$ months back in time.

Trail B is the only trail that requires that she step backward. For every 5 steps backward, she travels 7 months back in time. Therefore, for 25 steps backward, she used Trail B $25 \div 5 = 5$ times and travelled back in time $5 \times 7 = 35$ months.

She still needs to travel $60 - 35 = 25$ more months back in time. She has used Trail B 5 times, and since she uses the trails a total of 12 times, she has $12 - 5 = 7$ trail uses left. She can now only use Trail A and Trail C. We will present two solutions from this point.

Solution 1

If Gwen uses Trail A and Trail C one time each, she travels a total of 7 months back in time. If she uses Trail A and Trail C three times each, this accounts for six uses and she travels a total of $7 \times 3 = 21$ months back in time. She has one use left and still needs to travel 4 more months back in time. This can be accomplished by using Trail A once more.

It follows that Trail A is used 4 times and Trail C is used 3 times. The total number of forward steps is $4 \times 7 + 3 \times 2 = 28 + 6 = 34$.

Note that we could also have looked at each of the possibilities for using Trail A. Since there are a total of 7 trail uses for Trails A and C, the minimum number of uses for Trail A would be 0 and the maximum number of uses for Trail A would be 7. Once the number of uses for Trail A is selected, the number of uses for Trail C can be determined by subtracting the number of uses for Trail A from 7. For each combination we could determine the number of months travelled back in time. Once the correct combination is determined the total number of forward steps can be calculated. This is summarized in a table.

Uses of Trail A	Uses of Trail C	Months Travelled Back in Time
0	7	$0 \times 4 + 7 \times 3 = 0 + 21 = 21$
1	6	$1 \times 4 + 6 \times 3 = 4 + 18 = 22$
2	5	$2 \times 4 + 5 \times 3 = 8 + 15 = 23$
3	4	$3 \times 4 + 4 \times 3 = 12 + 12 = 24$
4	3	$4 \times 4 + 3 \times 3 = 16 + 9 = 25$
5	2	$5 \times 4 + 2 \times 3 = 20 + 6 = 26$
6	1	$6 \times 4 + 1 \times 3 = 24 + 3 = 27$
7	0	$7 \times 4 + 0 \times 3 = 28 + 0 = 28$

Only one combination gives the correct number of trail uses and the correct number of months travelled back in time. Using only Trail A and Trail C a total of 7 times, if we want to travel back in time 25 months we need to use Trail A 4 times and Trail C 3 times. The total number of forward steps is $4 \times 7 + 3 \times 2 = 28 + 6 = 34$.

Solution 2

This solution is presented for you to get a glimpse of what is coming in future mathematics courses.

Let a be the number of uses of Trail A, b be the number of uses of Trail B, and c be the number of uses of Trail C. Since the total number of uses is 12, then $a + b + c = 12$.

The total number of backward steps is 25 and Trail B is the only trail requiring backward steps. Since each use of Trail B requires 5 backward steps, then we require a total of 5 uses of Trail B to go back 25 steps. It follows that $b = 5$ and the equation $a + b + c = 12$ becomes $a + 5 + c = 12$, which simplifies to $a + c = 7$.

In using Trail B 5 times, Gwen travels a total of $5 \times 7 = 35$ months back in time. She needs to travel a total of 5 years or 60 months back in time. Thus, using Trail A and Trail C, she needs to travel $60 - 35 = 25$ more months back in time. Since she travels 4 months backward with each use of Trail A and 3 months backward with each use of Trail C, we need $4a + 3c = 25$.

Rearranging the equation $a + c = 7$, we obtain $c = 7 - a$. We can substitute for c in the equation $4a + 3c = 25$.

$$\begin{aligned} 4a + 3c &= 25 \\ 4a + 3(7 - a) &= 25 \\ 4a + 21 - 3a &= 25 \\ a + 21 &= 25 \\ a &= 4 \end{aligned}$$

We can substitute $a = 4$ into the equation $a + c = 7$ to determine that $c = 3$.

For each use of Trail A, 7 forward steps are required. Therefore, Gwen steps forward $7a$ steps using Trail A. For each use of Trail C, 2 forward steps are required. Therefore, Gwen steps forward $2c$ steps using Trail C. The total number of steps forward is $7a + 2c$. Since $a = 4$ and $c = 3$, the total number of forward steps is $7(4) + 2(3) = 28 + 6 = 34$.