Problem of the Week
Problem C and Solution
Two Numbers In

Problem

The POTW Input/Output Machine takes a number as input and adds 10 to the number. The machine then takes this sum and multiplies it by 2. Finally, the machine takes this product, subtracts 30 from the number, and outputs this new number.

Anala and Mei each input a positive integer into the machine. If the sum of their two outputs is 130, how many possibilities are there for the positive integer that Anala input into the machine?

INPUT → | Add 10 | → | Multiply by 2 | → | Subtract 30 | → OUTPUT

Solution

We will work backward from the final sum, 130, by 'undoing' each of the three operations to determine the sum of their two numbers before any operations were performed.

The final operation performed by the machine on each number was to subtract 30. Subtracting 30 from each number decreases their sum by 60. Therefore, the sum of the two numbers immediately before the third operation was performed was $130 + 60 = 190$.

Multiplying each of their numbers by 2 increases the sum of the two numbers by a factor of 2. Since the second sum of their two numbers was 190, the sum of their two numbers immediately before the second operation was performed must have been $180 \div 2 = 95$.

Finally, the first operation performed by each of Anala and Mei was to add 10 to their number. Adding 10 to each of their numbers increases the sum by 20, and so the sum of their numbers before the first operation must have been $95 - 20 = 75$.

Each of their original integers are positive and the two integers have a sum of 75.

Therefore, Anala's original integer could be any integer from 1 to 74, inclusive. Thus, there are 74 possibilities for Anala's original integer.