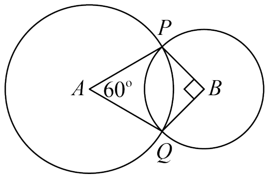


Problem of the Week

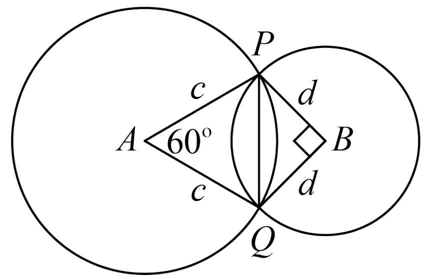
Problem D and Solution

The Other Area

Problem

Two circles, one with centre A and one with centre B, intersect at points P and Q such that $\angle PAQ = 60^\circ$ and $\angle PBQ = 90^\circ$. If the area of the circle with centre A is $48\,\text{m}^2$, what is the area of the circle with centre B?

Solution



Let c be the radius of the circle with centre A, in metres, and d be the radius of the circle with centre B, in metres. Then join P to Q.

We will determine the length of PQ in terms of c and then in terms of d in order to find a relationship between c and d.

Consider $\triangle APQ$. Since $AP = AQ = c$, $\triangle APQ$ is isosceles and so $\angle APQ = \angle AQP$. Since $\angle PAQ = 60^\circ$, $\angle APQ = \angle AQP = \frac{180^\circ - 60^\circ}{2} = 60^\circ$. Therefore, $\triangle APQ$ is equilateral and $PQ = AP = AQ = c$.

Consider $\triangle BPQ$. We are given that $\angle PBQ = 90^\circ$. Therefore, $\triangle BPQ$ is a right-angled triangle. The Pythagorean theorem tells us that $PQ^2 = BP^2 + BQ^2 = d^2 + d^2 = 2d^2$.

We have $PQ = c$ and $PQ^2 = 2d^2$. Therefore, $c^2 = 2d^2$.

The area of the circle with centre B and radius d is πd^2.

The area of the circle with centre A and radius c is πc^2. We know this area is equal to $48\,\text{m}^2$. Then,

$$\begin{aligned} 48 &= \pi c^2 \\ 48 &= \pi(2d^2) \\ 48 &= 2\pi d^2 \\ 24 &= \pi d^2 \end{aligned}$$

Therefore, the area of the circle with centre B is $24\,\text{m}^2$.