



## Problem of the Week

### Problem D and Solution

### How Many in the House?

#### Problem

The POTW Theatre has four levels of seating: gold, silver, red, and black.

One night, the manager of the theatre was asked how many patrons are in the theatre. The manager replied that  $\frac{1}{6}$  of the patrons in the theatre that night are in the gold seating,  $\frac{1}{4}$  of the patrons are in either the red seating or the black seating, there are three times as many patrons in the the silver seating as in the red seating, and there are 138 patrons in the black seating.

How many patrons were in the theatre that night?

#### Solution

Let  $n$  be the total number of patrons in the theatre that night.

Let  $g$  be the number of patrons in the gold seating,  $s$  be the number of patrons in the silver seating,  $r$  be the number of patrons in the red seating, and  $b$  be the number of patrons in the black seating.

Therefore,  $n = g + s + r + b$ .

Since  $\frac{1}{6}$  of the patrons in the theatre are in the gold seating,  $g = \frac{1}{6}n$ .

Since  $\frac{1}{4}$  of the patrons are either in the black seating or the red seating,  $r + b = \frac{1}{4}n$ .

It is given that  $b = 138$ . Therefore,  $r + b = \frac{1}{4}n$  becomes  $r + 138 = \frac{1}{4}n$ , or  $r = \frac{1}{4}n - 138$ .

Since there are three times as many patrons in the silver seating as patrons in the red seating,  $s = 3r = 3(\frac{1}{4}n - 138)$ .

Substituting these expressions for  $g$ ,  $r$ , and  $s$ , and the value for  $b$  into  $n = g + s + r + b$ , we have

$$\begin{aligned}n &= \left(\frac{1}{6}n\right) + 3\left(\frac{1}{4}n - 138\right) + \left(\frac{1}{4}n - 138\right) + 138 \\n &= \frac{1}{6}n + \frac{3}{4}n - 414 + \frac{1}{4}n - 138 + 138 \\n &= \frac{1}{6}n + n - 414 \\n &= \frac{7}{6}n - 414 \\\frac{1}{6}n &= 414 \\n &= 2484\end{aligned}$$

Therefore, there were 2484 patrons in the theatre that night.

Although it is not required, we could further determine that the number of patrons in the silver seating is 1449, the number of patrons in the gold seating is 414, and the number of patrons in the red seating is 483. We could then use these numbers to verify the given information.