



Problem of the Week

Problem D and Solution

Dye Refresher

Problem

To create an ink refresher for dye-based ink, some crafters will mix pure vegetable glycerine with water to get a mixture that is 12% vegetable glycerine, by volume. Kathy does not have pure vegetable glycerine, but she does have

- a 90 mL mixture that is 10.5% vegetable glycerine,
- a 120 mL mixture that is 30% vegetable glycerine, and
- a 1 L mixture that is 7.5% vegetable glycerine.

Since Kathy is a math teacher, she knows she can use the contents of these three mixtures to create a mixture that is 12% vegetable glycerine, by volume. She combines the contents of the entire 90 mL mixture with the contents of the entire 120 mL mixture, and then adds some of the 1 L mixture. How many millilitres of the 1 L mixture should she add to create a new mixture that is 12% vegetable glycerine, by volume?

Solution

Let x be the number of millilitres needed from the 1 L mixture.

The 90 mL mixture that is 10.5% vegetable glycerine has $0.105 \times 90 = 9.45$ mL of vegetable glycerine.

The 120 mL mixture that is 30% vegetable glycerine has $0.30 \times 120 = 36$ mL of vegetable glycerine.

In the x mL from the 1 L mixture, there is $0.075 \times x = 0.075x$ mL of vegetable glycerine.

Therefore, the total amount of vegetable glycerine in the new mixture is

$$9.45 + 36 + 0.075x = (45.45 + 0.075x) \text{ mL.}$$

The new mixture contains $90 + 120 + x = (210 + x)$ mL of liquid, of which 12% is vegetable glycerine.

Therefore, $0.12 \times (210 + x) = (25.2 + 0.12x)$ mL of the new mixture is vegetable glycerine.

Since we have shown that the amount of vegetable glycerine in the new mixture is $(45.45 + 0.075x)$ mL and $(25.2 + 0.12x)$ mL, we must have

$$\begin{aligned} 45.45 + 0.075x &= 25.2 + 0.12x \\ 0.075x - 0.12x &= 25.2 - 45.45 \\ -0.045x &= -20.25 \\ x &= 450 \end{aligned}$$

Therefore, she should add 450 mL of the 1 L mixture that is 7.5% vegetable glycerine.