Problem of the Week
Problem D and Solution
Again and Again and Again

Problem

When the fraction $\frac{1}{70\,000\,000}$ is written as a decimal, which digit occurs in the 2023$^{\text{rd}}$ place after the decimal point?

Solution

Notice that $\frac{1}{70\,000\,000} = \frac{1}{10\,000\,000} \times \frac{1}{7} = 0.000\,000\,1 \times \frac{1}{7}$.

Also, note that $\frac{1}{7} = 0.\overline{142857}$. That is, when $\frac{1}{7}$ is written as a decimal, the digits after the decimal point occur in repeating blocks of the 6 digits 142857.

Therefore,
$\frac{1}{70\,000\,000} = 0.000\,000\,1 \times \frac{1}{7} = 0.000\,000\,1 \times 0.\overline{142\,857} = 0.000\,000\,0\overline{14\,285\,7}$.
That is, when $\frac{1}{70\,000\,000}$ is written as a decimal, the digits after the decimal point will be seven zeros followed by repeating blocks of the six digits 142857.

We see the decimal representation of $\frac{1}{70\,000\,000}$ has the same repetition as that for $\frac{1}{7}$, but the pattern is shifted over 7 places. Since $2023 - 7 = 2016$, the 2023$^{\text{rd}}$ digit after the decimal point when $\frac{1}{70\,000\,000}$ is written as a decimal is the same as the 2016$^{\text{th}}$ digit after the decimal point when $\frac{1}{7}$ is written as a decimal.

Since $\frac{2016}{6} = 336$, then the 2016$^{\text{th}}$ digit after the decimal point occurs after exactly 336 repeating blocks of the 6 digits 142857. Therefore, the 2016$^{\text{th}}$ digit is the last digit in the repeating block, which is 7.

The 2023$^{\text{rd}}$ digit after the decimal point in the decimal representation of $\frac{1}{70\,000\,000}$ is the same as the 2016$^{\text{th}}$ digit after the decimal point in the decimal representation of $\frac{1}{7}$, and is therefore 7.