Problem of the Week

Problem D

How Many Fives?

The product of the first seven positive integers is equal to

$$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$$

Mathematicians will write this product as 7!. This is read as "7 factorial". So, $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$.

This factorial notation can be used with any positive integer. For example, $11! = 11 \times 10 \times 9 \times \cdots \times 3 \times 2 \times 1 = 39\,916\,800$. The three dots "$\cdots$" represent the product of the integers between 9 and 3.

Suppose $N = 1000!$. That is,

$$N = 1000! = 1000 \times 999 \times 998 \times 997 \times \cdots \times 3 \times 2 \times 1$$

Note that N is divisible by 5, 25, 125, and 625. Each of these factors is a power of 5. That is, $5 = 5^1$, $25 = 5^2$, $125 = 5^3$, and $625 = 5^4$.

Determine the largest power of 5 that divides N.