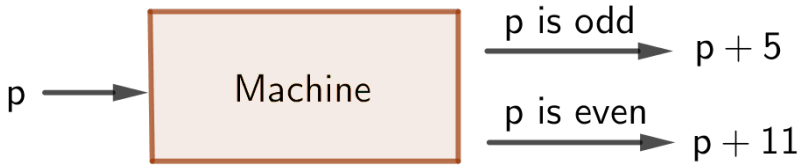
Problem of the Week
Problem D and Solution
Machine Math

Problem

A positive integer p is input into a machine. If p is odd, the machine outputs the integer $p + 5$. If p is even, the machine outputs the integer $p + 11$. This process can be repeated using each successive output as the next input. For example, if the input is $p = 1$ and the machine is used three times, the final output is 22.

If the input is $p = 2023$ and the machine is used 101 times, find the final output.



Solution

If p is odd, the output is $p + 5$, which is even because it is the sum of two odd integers. If p is even, the output is $p + 11$, which is odd, because it is the sum of an even integer and an odd integer.

Starting with $p = 2023$ and using the machine 2 times, we obtain $2023 + 5 = 2028$ and then $2028 + 11 = 2039$.

Starting with 2039 and using the machine 2 times, we obtain $2039 + 5 = 2044$ and then $2044 + 11 = 2055$.

Starting with an odd integer and using the machine 2 times, the net result is always adding 16 to the input, because the odd input generates a first output that is 5 larger (and so even) and a second output that is 11 larger than the first output. This generates a net result that is $5 + 11 = 16$ larger than the input.

Therefore, using the machine 96 more times (that is, repeating the 2 steps a total of 48 more times) we add 16 a total of 48 more times to obtain the output $2055 + 48 \times 16 = 2823$. To this point, the machine has been used 100 times.

The next time the machine is used, the output is $2823 + 5 = 2828$.

Thus, the final output after the machine is used 101 times is 2828.