# Problem of the Week
## Problem E and Solution
## A Pinch of Salt?

### Problem

A container initially contained 320 grams of common salt.

Mixture Y was formed by taking $x$ grams of the common salt out of the container, adding $x$ grams of Himalayan Pink Salt to the container, and then mixing uniformly. In Mixture Y, the ratio of the mass of the common salt to the mass of the Himalayan Pink Salt, expressed in lowest terms, was $c : h$.

Mixture Z was then formed by taking $x$ grams of Mixture Y out of the container, adding $x$ grams of Himalayan Pink Salt to the container, and then mixing uniformly. In Mixture Z, the ratio of the mass of the common salt to the mass of the Himalayan Pink Salt was $49 : 15$.

What is the value of $x + c + h$?

## Solution

Initially, the container contained 320 g of common salt and 0 g of Himalayan Pink Salt. Mixture Y contained $(320 - x)$ g of common salt and $x$ g of Himalayan Pink Salt. When Mixture Z (the final mixture) is formed, there was still a total of 320 g of salt in the bowl.

In Mixture Z, the ratio of the mass of common salt to the mass of Himalayan Pink Salt is $49 : 15$. Therefore, the mass of common salt is $\frac{49}{49+15} \cdot 320 = \frac{49}{64} \cdot 320 = 49 \cdot 5 = 245$ g and the mass of Himalayan Pink Salt in Mixture Z is $320 - 245 = 75$ g.

Mixture Y consisted of $(320 - x)$ g of common salt and $x$ g of Himalayan Pink Salt, which were thoroughly mixed together. Therefore, each gram of Mixture Y consisted of $\frac{320-x}{320}$ g of common salt and $\frac{x}{320}$ g of Himalayan Pink Salt.

To form Mixture Z, $x$ g of Mixture Y was removed. This amount of Mixture Y that was removed contained $x \cdot \frac{x}{320} = \frac{x^2}{320}$ g of Himalayan Pink Salt. Therefore, the mass of Himalayan Pink Salt in Mixture Z is the original $x$ g added to get Mixture Y minus the $\frac{x^2}{320}$ g from mixture Y plus the new $x$ g making $x - \frac{x^2}{320} + x = 2x - \frac{x^2}{320}$ g of Himalayan Pink Salt.

But we determined earlier that Mixture Z contains 75 g of Himalayan Pink Salt. Therefore,

$$\begin{aligned} 2x - \frac{x^2}{320} &= 75 \\ 0 &= x^2 - 2(320)x + 75(320) \\ 0 &= x^2 - 640x + 24\,000 \\ 0 &= (x - 40)(x - 600) \end{aligned}$$

Therefore, $x = 40$ or $x = 600$.
Since the initial mixture consisted of 320 g of common salt, then we must have $x < 320$. It follows that $x = 40$.
Therefore, Mixture Y consisted of $320 - 40 = 280$ g of common salt and 40 g of Himalayan Pink Salt. The ratio of these masses is $280 : 40$ or $7 : 1$ in lowest terms. Thus, $c = 7$ and $h = 1$.

Therefore, $x + c + h = 40 + 7 + 1 = 48$.