

Problem of the Week Problem E

A Scurry of Chipmunks

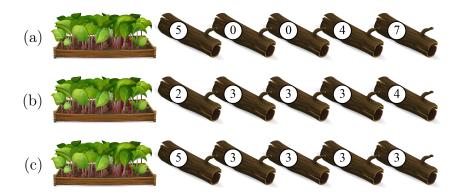
In Kumi's backyard there are five logs lined up in a row next to her vegetable garden. Unbeknown to her, under each log there is a burrow where some chipmunks live. The chipmunks do not always stay in the same burrow but they always stay in Kumi's backyard because they enjoy eating the food from her garden.

Every day each chipmunk counts the number of other chipmunks in their current burrow, as well as the number of chipmunks in the adjacent burrow(s). Every night, each chipmunk either stays in its current burrow, or moves to an adjacent burrow according to the following rules.

- 1. If the number of chipmunks in an adjacent burrow is less than the number of other chipmunks in their current burrow, then move to the adjacent burrow with the fewest number of chipmunks. In the case of a tie, move to the adjacent burrow that is closer to the garden.
- 2. Otherwise, if they are not already in the burrow closest to the garden, and the number of chipmunks in the adjacent burrow closer to the garden is equal to the number of other chipmunks in their current burrow, then move to the adjacent burrow closer to the garden.
- 3. Otherwise, stay in their current burrow.

The chipmunks follow these rules every day until all the chipmunks are in the same burrow; however this sometimes never happens!

For each of the following scenarios, determine whether or not all the chipmunks will end up in the same burrow, and if so, how many days it will take for this to happen. The number on each log represents the total number of chipmunks in the corresponding burrow on the initial day.



An example scenario and solution are given on the following page.

Consider the following example scenario.



We will look at what happens each night for the chipmunks in each burrow. We number the burrows from 1 to 5, starting with the burrow closest to the garden.

• Initial Day to Day 1:

- Each chipmunk in Burrow 1 currently has 5 others with them. Since there are fewer chipmunks in Burrow 2, all chipmunks in Burrow 1 will move to Burrow 2.
- Each chipmunk in Burrow 2 currently has 2 others with them. Since there are more chipmunks in Burrow 1, and the same amount in Burrow 3, all chipmunks in Burrow 2 will stay in Burrow 2.
- Each chipmunk in Burrow 3 currently has 1 other with them. Since there are fewer chipmunks in Burrow 4, all chipmunks in Burrow 3 will move to Burrow 4.
- Each chipmunk in Burrow 5 currently has 3 others with them. Since there are fewer chipmunks in Burrow 4, all chipmunks in Burrow 5 will move to Burrow 4.

The chipmunks will then be arranged as follows: GARDEN 0 9 0 6 0

• Day 1 to Day 2:

- Each chipmunk in Burrow 2 currently has 8 others with them. Since there are 0 chipmunks in Burrows 1 and 3, all chipmunks in Burrow 2 will move to Burrow 1, because it is closer to the garden.
- Each chipmunk in Burrow 4 currently has 5 others with them. Since there are 0 chipmunks in Burrows 3 and 5, all chipmunks in Burrow 4 will move to Burrow 3, because it is closer to the garden.

The chipmunks will then be arranged as follows: GARDEN 9 0 6 0 0

• Day 2 to Day 3:

- Each chipmunk in Burrow 1 currently has 8 others with them. Since there are fewer chipmunks in Burrow 2, all chipmunks in Burrow 1 will move to Burrow 2.
- Each chipmunk in Burrow 3 currently has 5 others with them. Since there are 0 chipmunks in Burrows 2 and 4, all chipmunks in Burrow 3 will move to Burrow 2, because it is closer to the garden.

The chipmunks will then be arranged as follows: GARDEN 0 15 0 0 0

Therefore, after 3 days, all the chipmunks will be in the same burrow.