



Problem of the Week

Problem E and Solution

A Scurry of Chipmunks

Problem

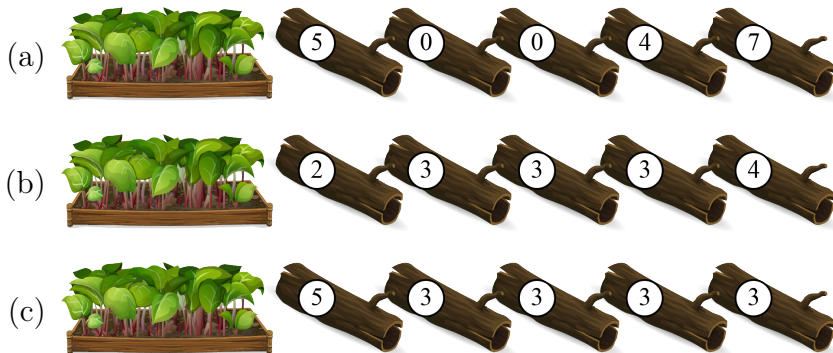
In Kumi's backyard there are five logs lined up in a row next to her vegetable garden. Unbeknownst to her, under each log there is a burrow where some chipmunks live. The chipmunks do not always stay in the same burrow but they always stay in Kumi's backyard because they enjoy eating the food from her garden.

Every day each chipmunk counts the number of *other* chipmunks in their current burrow, as well as the number of chipmunks in the adjacent burrow(s). Every night, each chipmunk either stays in its current burrow, or moves to an adjacent burrow according to the following rules.

1. If the number of chipmunks in an adjacent burrow is less than the number of *other* chipmunks in their current burrow, then move to the adjacent burrow with the fewest number of chipmunks. In the case of a tie, move to the adjacent burrow that is closer to the garden.
2. Otherwise, if they are not already in the burrow closest to the garden, and the number of chipmunks in the adjacent burrow closer to the garden is equal to the number of *other* chipmunks in their current burrow, then move to the adjacent burrow closer to the garden.
3. Otherwise, stay in their current burrow.

The chipmunks follow these rules every day until all the chipmunks are in the same burrow; however this sometimes never happens!

For each of the following scenarios, determine whether or not all the chipmunks will end up in the same burrow, and if so, how many days it will take for this to happen. The number on each log represents the total number of chipmunks in the corresponding burrow on the initial day.





Solution

We number the burrows from 1 to 5, starting with the burrow closest to the garden. To help condense the solution, we have written these as B1, B2, B3, B4, and B5.

(a) We will look at what happens each night for the chipmunks in each burrow.

- Initial Day to Day 1:
 - All chipmunks in B1 move to B2, since 0 others is less than 4 others.
 - All chipmunks in B4 move to B3, since 0 others is less than 3 others or 7 others.
 - All chipmunks in B5 move to B4, since 4 others is less than 6 others.

The chipmunks will then be arranged as follows: GARDEN 0 5 4 7 0

- Day 1 to Day 2:
 - All chipmunks in B2 move to B1, since 0 others is less than 4 others.
 - All chipmunks in B3 stay in B3, since 3 others is less than 5 others or 7 others.
 - All chipmunks in B4 move to B5, since 0 others is less than 6 others or 4 others.

The chipmunks will then be arranged as follows: GARDEN 5 0 4 0 7

- Day 2 to Day 3:
 - All chipmunks in B1 move to B2, since 0 others is less than 4 others.
 - All chipmunks in B3 move to B2, since 0 others is less than 3 others, and B2 is closer to the garden than B4.
 - All chipmunks in B5 move to B4, since 0 others is less than 6 others.

The chipmunks will then be arranged as follows: GARDEN 0 9 0 7 0

- Day 3 to Day 4:
 - All chipmunks in B2 move to B1, since 0 others is less than 8 others, and B1 is closer to the garden than B3.
 - All chipmunks in B4 move to B3, since 0 others is less than 6 others, and B3 is closer to the garden than B5.

The chipmunks will then be arranged as follows: GARDEN 9 0 7 0 0

- Day 4 to Day 5:
 - All chipmunks in B1 move to B2, since 0 others is less than 8 others.
 - All chipmunks in B3 move to B2, since 0 others is less than 6 others, and B2 is closer to the garden than B4.

The chipmunks will then be arranged as follows: GARDEN 0 16 0 0 0

Therefore, after 5 days, all the chipmunks will be in the same burrow.

(b) We will look at what happens each night for the chipmunks in each burrow.

- Initial Day to Day 1:
 - All chipmunks in B1 stay in B1, since 1 other is less than 3 others.
 - All chipmunks in B2 move to B1, since B1 is closer to the garden.
 - All chipmunks in B3 stay in B3, since 2 others is less than 3 others.
 - All chipmunks in B4 stay in B4, since 2 others is less than 3 others or 4 others.
 - All chipmunks in B5 move to B4, since B4 is closer to the garden.

The chipmunks will then be arranged as follows: GARDEN 5 0 3 7 0



- Day 1 to Day 2:
 - All chipmunks in B1 move to B2, since 0 others is less than 4 others.
 - All chipmunks in B3 move to B2, since 0 others is less than 2 others or 7 others.
 - All chipmunks in B4 move to B5, since 0 others is less than 6 others or 3 others.

The chipmunks will then be arranged as follows: GARDEN 0 8 0 0 7

- Day 2 to Day 3:
 - All chipmunks in B2 move to B1, since 0 others is less than 7 others, and B1 is closer to the garden than B3.
 - All chipmunks in B5 move to B4, since 0 others is less than 6 others.

The chipmunks will then be arranged as follows: GARDEN 8 0 0 7 0

- Day 3 to Day 4:
 - All chipmunks in B1 move to B2, since 0 others is less than 7 others.
 - All chipmunks in B4 move to B3, since 0 others is less than 6 others, and B3 is closer to the garden than B5.

The chipmunks will then be arranged as follows: GARDEN 0 8 7 0 0

- Day 4 to Day 5:
 - All chipmunks in B2 move to B1, since 0 others is less than 7 others.
 - All chipmunks in B3 move to B4, since 0 others is less than 6 others or 8 others.

The chipmunks will then be arranged as follows: GARDEN 8 0 0 7 0

This is the same as the start of Day 3, so Day 6 will start the same as Day 4, and the cycle will repeat. Therefore, all the chipmunks will never end up in the same burrow.

(c) We will look at what happens each night for the chipmunks in each burrow.

- Initial Day to Day 1:
 - All chipmunks in B1 move to B2, since 3 others is less than 4 others.
 - All chipmunks in B2 stay in B2, since 2 others is less than 3 others or 5 others.
 - All chipmunks in B3, B4, and B5 stay in their current burrows, since 2 others is less than 3 others.

The chipmunks will then be arranged as follows: GARDEN 0 8 3 3 3

- Day 1 to Day 2:
 - All chipmunks in B2 move to B1, since 0 others is less than 7 others or 3 others.
 - All chipmunks in B3 stay in B3, since 2 others is less than 8 others or 3 others.
 - All chipmunks in B4 and B5 stay in their current burrows, since 2 others is less than 3 others.

The chipmunks will then be arranged as follows: GARDEN 8 0 3 3 3

- Day 2 to Day 3:
 - All chipmunks in B1 move to B2, since 0 others is less than 7 others.
 - All chipmunks in B3 move to B2, since 0 others is less than 2 others or 3 others.
 - All chipmunks in B4 and B5 stay in their current burrows, since 2 others is less than 3 others.

The chipmunks will then be arranged as follows: GARDEN 0 11 0 3 3



- Day 3 to Day 4:
 - All chipmunks in B2 move to B1, since 0 others is less than 10 others, and B1 is closer to the garden than B3.
 - All chipmunks in B4 move to B3, since 0 others is less than 2 others or 3 others.
 - All chipmunks in B5 stay in B5, since 2 others is less than 3 others.

The chipmunks will then be arranged as follows:

GARDEN

 11 0 3 0 3

- Day 4 to Day 5:
 - All chipmunks in B1 move to B2, since 0 others is less than 10 others.
 - All chipmunks in B3 move to B2, since 0 others is less than 2 others, and B2 is closer to the garden than B4.
 - All chipmunks in B5 move to B4, since 0 others is less than 2 others.

The chipmunks will then be arranged as follows:

GARDEN

 0 14 0 3 0

- Day 5 to Day 6:
 - All chipmunks in B2 move to B1, since 0 others is less than 13 others, and B1 is closer to the garden than B3.
 - All chipmunks in B4 move to B3, since 0 others is less than 2 others, and B3 is closer to the garden than B5.

The chipmunks will then be arranged as follows:

GARDEN

 14 0 3 0 0

- Day 6 to Day 7:
 - All chipmunks in B1 move to B2, since 0 others is less than 13 others.
 - All chipmunks in B3 move to B2, since 0 others is less than 2 others, and B2 is closer to the garden than B4.

The chipmunks will then be arranged as follows:

GARDEN

 0 17 0 0 0

Therefore, after 7 days, all the chipmunks will be in the same burrow.

EXTENSION:

In parts (a) and (c), the chipmunks all came together in Burrow 2. If the chipmunks do come together in the same burrow, will it always happen in Burrow 2?