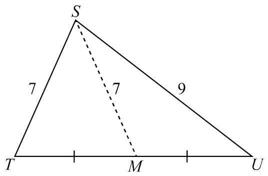


Problem of the Week

Problem E and Solution

The Other Side

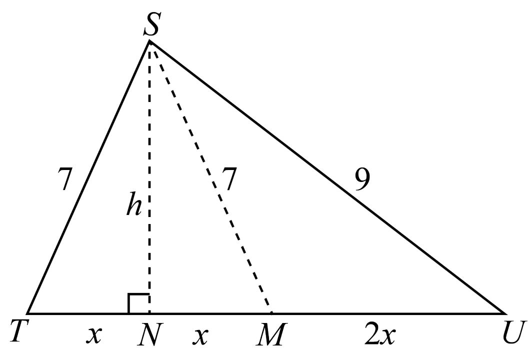
Problem

In $\triangle STU$, a median is drawn from vertex S, meeting side TU at point M. The length of side ST is 7 cm, the length of side SU is 9 cm, and the length of the median SM is 7 cm. Determine the length of TU.

Solution

Solution 1

Since $ST = SM = 7$, $\triangle STM$ is isosceles. In $\triangle STM$, draw an altitude from vertex S to TM, intersecting TM at N. Let $TN = x$. In an isosceles triangle, the altitude drawn to the base bisects the base. Therefore, $NM = TN = x$. Since SM is a median in $\triangle STU$, it follows that $MU = TM = 2x$. Let $SN = h$.



Since $\triangle SNM$ is a right-angled triangle, we can use the Pythagorean Theorem as follows.

$$\begin{aligned} SN^2 &= SM^2 - NM^2 \\ h^2 &= 7^2 - x^2 \\ h^2 &= 49 - x^2 \qquad (1) \end{aligned}$$

Since $\triangle SNU$ is a right-angled triangle, we can use the Pythagorean Theorem as follows.

$$\begin{aligned} SN^2 &= SU^2 - NU^2 \\ h^2 &= 9^2 - (x + 2x)^2 \\ h^2 &= 81 - (3x)^2 \\ h^2 &= 81 - 9x^2 \qquad (2) \end{aligned}$$

In both equations (1) and (2), the left side is h^2. Therefore, the right side of equation (1) must equal the right side of equation (2).

$$\begin{aligned} 49 - x^2 &= 81 - 9x^2 \\ -x^2 + 9x^2 &= 81 - 49 \\ 8x^2 &= 32 \\ x^2 &= 4 \end{aligned}$$

Since $x > 0$, it follows that $x = 2$.

Therefore, $TU = TN + NM + MU = x + x + 2x = 4x = 4(2) = 8$ cm.

Solution 2

This solution is presented for students who have done some trigonometry and know the Cosine Law. Since SM is a median, let $TM = MU = y$. Then $TU = 2y$.

Using the Cosine Law in $\triangle STM$,

$$\begin{aligned} SM^2 &= ST^2 + TM^2 - 2(ST)(TM)\cos T \\ 7^2 &= 7^2 + y^2 - 2(7)(y)\cos T \\ 49 &= 49 + y^2 - 14y\cos T \\ 14y\cos T &= y^2 \qquad (1) \end{aligned}$$

Using the Cosine Law in $\triangle STU$,

$$\begin{aligned} SU^2 &= ST^2 + TU^2 - 2(ST)(TU)\cos T \\ 9^2 &= 7^2 + (2y)^2 - 2(7)(2y)\cos T \\ 81 &= 49 + 4y^2 - 28y\cos T \\ 28y\cos T &= 4y^2 - 32 \\ 14y\cos T &= 2y^2 - 16 \qquad (2) \end{aligned}$$

Subtracting equation (2) from equation (1) allows us to solve for y.

$$\begin{aligned} 14y\cos T &= y^2 \qquad (1) \\ 14y\cos T &= 2y^2 - 16 \qquad (2) \\ 0 &= -y^2 + 16 \\ y^2 &= 16 \end{aligned}$$

Since $y > 0$, it follows that $y = 4$.

Therefore, the length of TU is $2(4) = 8$ cm.