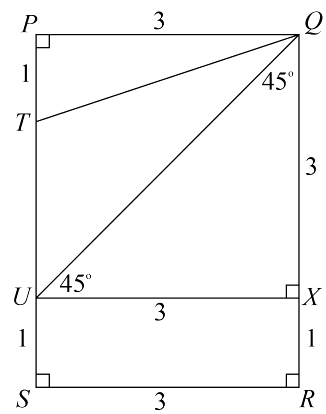
Problem of the Week
Problem E and Solution
The Angle Between

Problem

Rectangle $PQRS$ has $PQ = 3$ and $QR = 4$. Points T and U are on side PS such that $PT = US = 1$. Determine the measure of $\angle TQU$, in degrees and rounded to 1 decimal place.

Solution

Let X be the point on QR such that UX is parallel to SR. Then $\angle UXQ = 90°$. Also, $UX = SR = 3$, $XR = US = 1$, and therefore $QX = 3$.
It follows that $\triangle UXQ$ is an isosceles right-angled triangle, and so $\angle UQX = \angle QUX = 45°$.
From here we present three different solutions.



Solution 1

Since $\triangle TPQ$ is a right-angled triangle, $\tan(\angle TQP) = \frac{1}{3}$, and so $\angle TQP \approx 18.4°$.

Since $\angle PQX = 90°$, we can calculate the value of $\angle TQU$ as follows.

$$\begin{aligned}\angle PQX &= \angle PQT + \angle TQU + \angle UQX \\ \angle TQU &= \angle PQX - \angle PQT - \angle UQX \\ \angle TQU &\approx 90° - 18.4° - 45° \\ \angle TQU &\approx 26.6°\end{aligned}$$

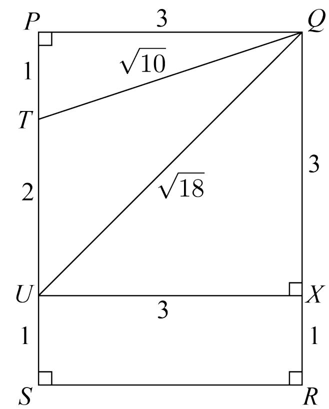
Therefore, $\angle TQU \approx 26.6°$.

Solution 2

Since $\triangle TPQ$ is a right-angled triangle, by the Pythagorean Theorem, $TQ^2 = PT^2 + PQ^2 = 1^2 + 3^2 = 10$. Therefore $TQ = \sqrt{10}$, since $TQ > 0$.

Since $PQRS$ is a rectangle, $PS = QR = 4$, and $PT = US = 1$, it follows that $TU = 2$. Since $\triangle UXQ$ is a right-angled triangle, by the Pythagorean Theorem, $QU^2 = UX^2 + QX^2 = 3^2 + 3^2 = 18$. Therefore $QU = \sqrt{18}$, since $QU > 0$. Now we will use the cosine law in $\triangle TQU$.

$$\begin{aligned}
TU^2 &= TQ^2 + QU^2 - 2(TQ)(QU)\cos(\angle TQU) \\
2^2 &= 10 + 18 - 2(\sqrt{10})(\sqrt{18})\cos(\angle TQU) \\
4 - 10 - 18 &= -2\sqrt{10}\sqrt{18}\cos(\angle TQU) \\
-24 &= -2\sqrt{10}\sqrt{18}\cos(\angle TQU) \\
\frac{12}{\sqrt{10}\sqrt{18}} &= \cos(\angle TQU) \\
\angle TQU &\approx 26.6^\circ
\end{aligned}$$



Therefore, $\angle TQU \approx 26.6^\circ$.

Solution 3

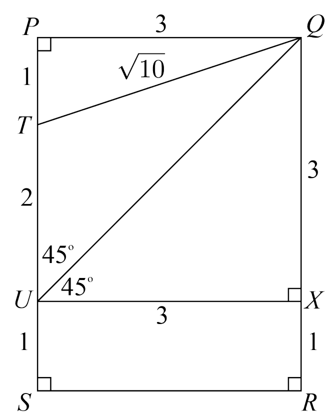
Since $\triangle TPQ$ is a right-angled triangle, by the Pythagorean Theorem, $TQ^2 = PT^2 + PQ^2 = 1^2 + 3^2 = 10$. Therefore $TQ = \sqrt{10}$, since $TQ > 0$.

Since $PQRS$ is a rectangle, $PS = QR = 4$, and $PT = US = 1$, it follows that $TU = 2$.

Since UX is parallel to SR, then $\angle PUX = 90^\circ$. Since $\angle QUX = 45^\circ$, it follows that $\angle TUQ = 45^\circ$.

Now we will use the sine law in $\triangle TQU$.

$$\begin{aligned}
\frac{\sin(\angle TQU)}{TU} &= \frac{\sin(\angle TUQ)}{TQ} \\
\frac{\sin(\angle TQU)}{2} &= \frac{\sin 45^\circ}{\sqrt{10}} \\
\angle TQU &\approx 26.6^\circ
\end{aligned}$$



Therefore, $\angle TQU \approx 26.6^\circ$.