



## Problem of the Week

### Problem E and Solution

### Two Digits at a Time

#### Problem

The number 34692 contains 5 digits, and so we say its digit length is 5. The first digit of 34692 is 3. Also, the two-digit integers formed by choosing any pair of consecutive digits, that is, 34, 46, 69, and 92, are all divisible by either 17 or 23.

An integer with digit length 2022 has first digit 3. This integer also has the property that the two-digit integers formed by choosing any pair of consecutive digits are all divisible by either 17 or 23.

List all of the possibilities for the last three digits of this integer.

#### Solution

We first list out all two-digit multiples of 17 and 23.

The two-digit multiples of 17 are 17, 34, 51, 68, and 85.

The two-digit multiples of 23 are 23, 46, 69, and 92.

Since the integer starts with a 3 and the only two-digit number in the two lists that starts with a 3 is 34, the second digit must be 4. Similarly, the third digit is 6, since the only two-digit number in the two lists starting with a 4 is 46. However, the fourth digit can be 8 or 9, since there are two two-digit numbers, 68 and 69, in the two lists that start with a 6. Now we need to consider two cases.

- Case 1: The fourth digit is an 8.

Since the only two-digit number in the two lists starting with an 8 is 85, the fifth digit must then be 5. Similarly, the sixth digit is 1, and the seventh digit is 7. We must stop here, since there is no two-digit number in either list that starts with a 7. Therefore, the fourth digit must not be 8.

- Case 2: The fourth digit is 9.

Since the only two-digit number in the two lists starting with a 9 is 92, the fifth digit must then be 2. Similarly, the sixth digit is 3. We can now repeat our argument from the beginning. The digit after the 3 must be a 4, then a 6, then a 9, then a 2. The five digits '34692' will continue to repeat as long as a 9 follows the 6.

Since  $2022 \div 5 = 404.4$ , the number of length 2022 will consist of 404 blocks of five digits followed by two more digits. As we saw above, the first 403 blocks of digits must each be the five digits 34692. However, the last seven digits could be 3469234 or 3468517.

Therefore, there are two possibilities for the last three digits. The last three digits could be 234 or 517.