Problem of the Week  
Problem C and Solution  
Overlapping Shapes 1

Problem
Omar draws square $ABCD$ with side length 4 cm. Jaime then draws $\triangle AED$ on top of square $ABCD$ so that

- sides $AE$ and $DE$ meet $BC$ at $F$ and $G$, respectively,
- $FG$ is 3 cm, and
- the area of $\triangle AED$ is twice the area of square $ABCD$.

Determine the area of $\triangle FEG$.

Solution
Solution 1
In the first solution we will find the area of square $ABCD$, the area of $\triangle AED$, the area of trapezoid $AFGD$, and then use these to calculate the area of $\triangle FEG$.

![Diagram of overlapping shapes]

The area of square $ABCD$ is $4 \times 4 = 16$ cm$^2$. Since the area of $\triangle AED$ is twice the area of square $ABCD$, it follows that the area of $\triangle AED$ is $2 \times 16 = 32$ cm$^2$.

Recall that to find the area of a trapezoid, we multiply the sum of the lengths of the two parallel sides by the height, and divide the product by 2. In trapezoid $AFGD$, the two parallel sides are $AD$ and $FG$, and the height is the width of square $ABCD$, namely $AB$.

Area of trapezoid $AFGD = (AD + FG) \times AB \div 2$

$= (4 + 3) \times 4 \div 2$

$= 7 \times 4 \div 2$

$= 14$ cm$^2$

The area of $\triangle FEG$ is equal to the area of $\triangle AED$ minus the area of trapezoid $AFGD$. Thus, the area of $\triangle FEG$ is $32 - 14 = 18$ cm$^2$. 
Solution 2

We construct an altitude of \(\triangle AED\) from \(E\), intersecting \(AD\) at \(P\) and \(BC\) at \(Q\). Since \(ABCD\) is a square, we know that \(AD\) is parallel to \(BC\). Therefore, since \(PE\) is perpendicular to \(AD\), \(QE\) is perpendicular to \(FG\) and thus an altitude of \(\triangle FEG\). In this solution we will find the height of \(\triangle FEG\), that is, the length of \(QE\), and then use this to calculate the area of \(\triangle FEG\).

The area of square \(ABCD\) is \(4 \times 4 = 16\ \text{cm}^2\). Since the area of \(\triangle AED\) is twice the area of square \(ABCD\), it follows that the area of \(\triangle AED\) is \(2 \times 16 = 32\ \text{cm}^2\).

We also know that

\[
\text{Area } \triangle AED = AD \times PE \div 2
\]

\[
32 = 4 \times PE \div 2
\]

\[
32 = 2 \times PE
\]

\[
PE = 32 \div 2
\]

\[
= 16\ \text{cm}
\]

Since \(\angle APQ = 90^\circ\), we know that \(ABQP\) is a rectangle, and so \(PQ = AB = 4\ \text{cm}\). We also know that \(PE = PQ + QE\). Since \(PE = 16\ \text{cm}\) and \(PQ = 4\ \text{cm}\), it follows that

\[
QE = PE - PQ = 16 - 4 = 12\ \text{cm}
\]

We can then calculate the area of \(\triangle FEG\).

\[
\text{Area } \triangle FEG = FG \times QE \div 2
\]

\[
= 3 \times 12 \div 2
\]

\[
= 18\ \text{cm}^2
\]

Therefore, the area of \(\triangle FEG\) is \(18\ \text{cm}^2\).