Problem of the Week
Problem C and Solution
Sharing Grapes

Problem
Jessica has some grapes. She gives one-third of her grapes to Callista. She then gives 4 grapes to Monica. Finally, she gives one-half of her remaining grapes to Peter. If Jessica then has 16 grapes left, how many grapes did Jessica begin with?

Solution
Solution 1:
We work backwards from the last piece of information given.
Jessica has 16 grapes left after giving one-half of her remaining grapes to Peter. This means that she had \(2 \times 16 = 32\) grapes immediately before giving grapes to Peter.
Immediately before giving grapes to Peter, she gave 4 grapes to Monica, which means that she had \(32 + 4 = 36\) grapes immediately before giving 4 grapes to Monica.
Immediately before giving the 4 grapes to Monica, she gave one-third of her grapes to Callista, which would have left her with two-thirds of her original amount.
Since two-thirds of her original amount equals 36 grapes, then one-third equals one half of 36 or \(\frac{36}{2} = 18\) grapes.
Thus, she gave 18 grapes to Callista, and so Jessica began with \(36 + 18 = 54\) grapes.

Solution 2:
Suppose Jessica started with \(x\) grapes.
She gives \(\frac{1}{3}x\) grapes to Callista, leaving her with \(1 - \frac{1}{3}x = \frac{2}{3}x\) grapes.
She then gives 4 grapes to Monica, leaving her with \(\frac{2}{3}x - 4\) grapes.
Finally, she gives away one-half of what she has left to Peter, which means that she keeps one-half of what she has left, and so she keeps \(\frac{1}{2}(\frac{2}{3}x - 4)\) grapes.
Simplifying this expression, we obtain \(\frac{2}{6}x - \frac{4}{2} = \frac{1}{3}x - 2\) grapes.
Since she has 16 grapes left, then \(\frac{1}{3}x - 2 = 16\) and so \(\frac{1}{3}x = 18\) or \(x = 54\).
Therefore, Jessica began with 54 grapes.