# Problem of the Week
Problem C and Solution
Take a Seat 1

Problem

Twelve people are seated around a circular table. They each hold a card with a different integer from 1 to 12 on it. For any two people sitting beside each other, the positive difference between the integers on their cards is no more than 2. The people with integers 1, 3, a, and b are seated as shown.

What is the value of $a + b$?
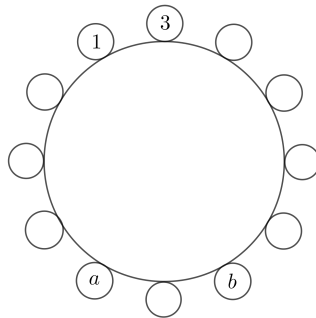
Solution

Because two integers that are beside each other must have a positive difference of at most 2, then the possible neighbours of 1 are 2 and 3. Since 1 has exactly two neighbours, then 1 must be between 2 and 3.

Next, consider 2. Its possible neighbours are 1, 3, and 4. The number 2 is already a neighbour of 1 and cannot be a neighbour of 3 (since 3 is on the other side of 1). Therefore, 2 is between 1 and 4. This allows us to update the diagram as shown.

Continuing in this way, the possible neighbours of 3 are 1, 2, 4, and 5. The number 1 is already beside 3, and the numbers 2 and 4 cannot be beside 3. So 5 must be beside 3.

The possible neighbours of 4 are 2, 3, 5, and 6. The number 2 is already beside 4. Numbers 3 and 5 cannot be beside 4. So 6 must be beside 4.

Similarly, we know 7 will be beside 5 and 8 will be beside 6. Thus, $a = 8$. Continuing this way, we know 9 is beside 7, 10 is beside 8, 11 is beside 9, and 12 is beside 10. Thus, $b = 12$. The completed circle is shown.

Therefore, $a + b = 8 + 12 = 20$.