Problem of the Week

Problem C and Solution

Transformational Moves

Problem

The three points $A(1, 1)$, $B(1, 4)$, and $C(2, 1)$ are the vertices of $\triangle ABC$. We perform transformations to the triangle, as follows. First, we shift $\triangle ABC$ to the right 4 units. Then, we reflect the image in the $x$-axis. Then, we reflect the new image in the $y$-axis. Finally, we shift the newest image up 5 units.

What are the coordinates of the vertices of the final triangle?

Solution

In the solution we are going to use notation that is commonly used in transformations. When we transform point $A$, we label the transformed point as $A'$. We call this “$A$ prime”. When we transform point $A'$, we label the transformed point as $A''$. We call this “$A$ double prime”. This can continue for all four transformations and for vertices $B$ and $C$ as well.

When $\triangle ABC$ is shifted to the right 4 units, the $x$-coordinate of each vertex increases by 4. Thus, $\triangle A'B'C'$ has vertices $A'(5, 1), B'(5, 4),$ and $C'(6, 1)$.

When $\triangle A'B'C'$ is reflected in the $x$-axis, we multiply the $y$-coordinate of each vertex by $-1$. Thus, $\triangle A''B''C''$ has vertices $A''(5, -1), B''(5, -4),$ and $C''(6, -1)$. 
When $\triangle A''B''C''$ is reflected in the $y$-axis, we multiply the $x$-coordinate of each vertex by $-1$. Thus, $\triangle A'''B'''C'''$ has vertices $A'''(-5,-1)$, $B'''(-5,-4)$, and $C'''(-6,-1)$.

When $\triangle A'''B'''C'''$ is shifted up 5 units, the $y$-coordinate of each vertex increases by 5. Thus, $\triangle A''''B''''C''''$ has vertices $A''''(-5,4)$, $B''''(-5,1)$, and $C''''(-6,4)$.

Thus, the final triangle has vertices $A''''(-5,4)$, $B''''(-5,1)$ and $C''''(-6,4)$. 