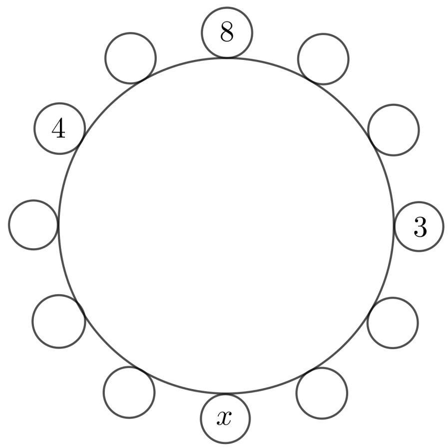
Problem of the Week

Problem D and Solution

Take a Seat 2

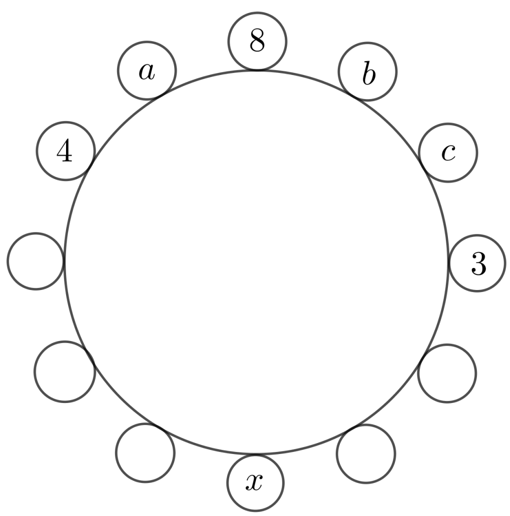
Problem

Twelve people are seated, equally spaced, around a circular table. They each hold a card with different integer on it. For any two people sitting beside each other, the positive difference between the integers on their cards is no more than 2. The people holding the integers 3, 4, and 8 are seated as shown. The person opposite the person holding 8 is holding the integer x. What are the possible values of x?

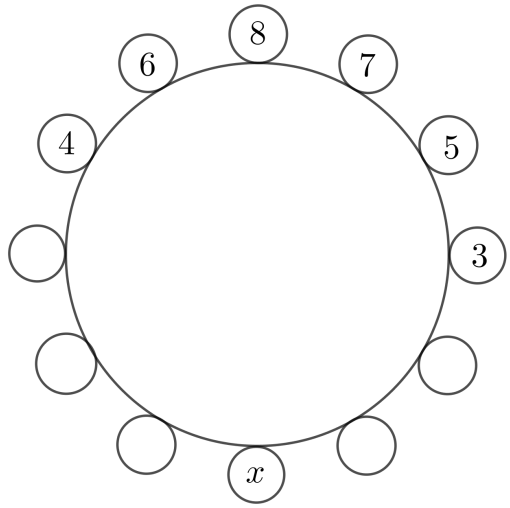


Solution

Let a represent the integer on the card between the card numbered 4 and the card numbered 8, and let b and c represent the integers on the cards between the card numbered 8 and the card numbered 3, as shown in the diagram.



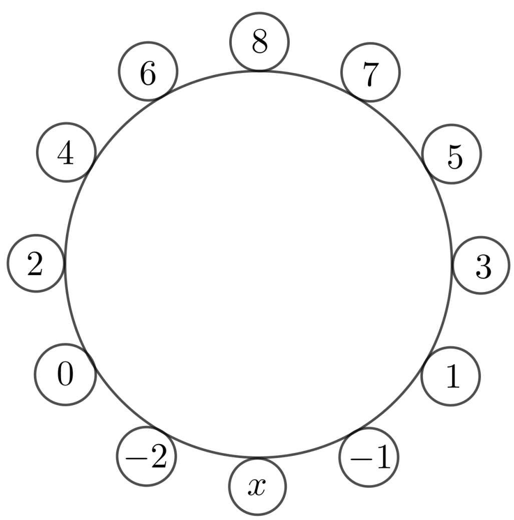
The integer 6 is the only integer that is within 2 of both 4 and 8. Therefore, $a = 6$. Now, b can be either 7, 9, or 10. (We cannot have $b = 6$ since each person has a card with a different integer on it.) If $b = 9$ or $b = 10$, then for c there is no integer that is within 2 of b and 3. Therefore, $b = 7$. Furthermore, the integer 5 is the only integer that is within 2 of both 3 and 7. Therefore, $c = 5$.



Next, we again consider the card numbered 4. The possible card numbers for its neighbours are 2, 3, 5, and 6. It is already beside the card numbered 6, and the integers 3 and 5 are on cards that are not beside the card numbered 4. Therefore, the card on the other side of the card numbered 4 must be numbered 2.

Next, we again consider the card numbered 3. The possible card numbers for its neighbours are 1, 2, 4, and 5. It is already beside the card numbered 5, and the integers 2 and 4 are on cards that are not beside the card numbered 3. Therefore, the card on the other side of the card numbered 3 must be numbered 1.

We continue in this way to determine that the other card beside the card numbered 2 must be numbered 0. Then, the other card beside the card numbered 1 must be numbered -1. Then, the other card beside the card numbered 0 must be numbered -2.



Finally, the possible card numbers for the neighbours of the card numbered -2 are 0, -1, -3, and -4. Also, the possible card numbers for the neighbours of the card numbered -1 are 1, 0, -2, and -3. Thus, since x is a neighbour of both the card numbered -2 and the card numbered -1, we must have $x = 0$ or $x = -3$. Since 0 is already on another card, then $x = -3$.