Problem of the Week
Problem D and Solution
Throw to Win

Problem
Kurtis is creating a game for a math fair. They attach \(n\) circles, each with radius 1 metre, onto a square wall with side length \(n\) metres, where \(n\) is a positive integer, so that none of the circles overlap. Participants will throw a dart at the wall and if the dart lands on a circle, they win a prize. Kurtis wants the probability of winning the game to be at least \(\frac{1}{2}\).

If they assume that each dart hits the wall at a single random point, then what is the largest possible value of \(n\)?

Solution
The area of the square wall with side length \(n\) metres is \(n^2\) square metres.

The area of each circle is \(\pi(1)^2 = \pi\) square metres. Since there are \(n\) circles, the total area covered by circles is \(n\pi\) square metres.

If each dart hits the wall at a single random point, then the probability that a dart lands on a circle is equal to the area of the wall covered by circles divided by the total area of the wall. That is,

\[
\frac{n\pi \text{ square metres}}{n^2 \text{ square metres}} = \frac{\pi}{n}
\]

If this probability must be at least \(\frac{1}{2}\), then

\[
\frac{\pi}{n} \geq \frac{1}{2}
\]

\[
\pi \geq \frac{n}{2}, \quad \text{since } n > 0
\]

\[
2\pi \geq n
\]

\[
n \leq 2\pi \approx 6.28
\]

Thus, since \(n\) is an integer, the largest possible value of \(n\) is 6.