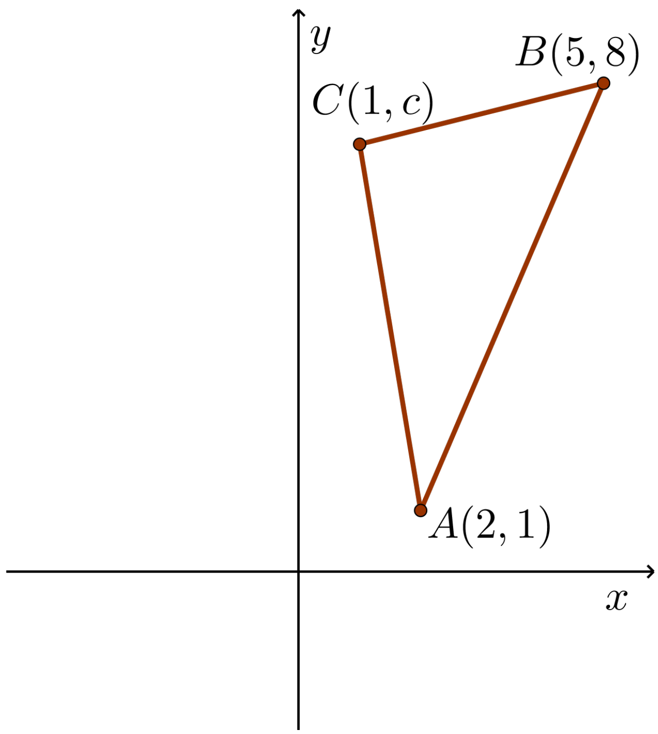
Problem of the Week
Problem D and Solution
Reflect on This

Problem

$\triangle ABC$ has vertices $A(2,1)$, $B(5,8)$, and $C(1,c)$, where $c > 0$.

Vertices A and B are reflected in the y-axis, and vertex C is reflected in the x-axis. The three image points are collinear. That is, a line passes through the three image points.

Determine the coordinates of C.



Solution

When a point is reflected in the y-axis, the image point has the same y-coordinate and the x-coordinate is -1 multiplied by the pre-image x-coordinate. Thus, the image of $A(2,1)$ is $A'(-2,1)$, and the image of $B(5,8)$ is $B'(-5,8)$.

When a point is reflected about the x-axis, the image point has the same x-coordinate and the y-coordinate is -1 multiplied by the pre-image y-coordinate. Thus, the image of $C(1,c)$ is $C'(1,-c)$.

The three image points, A', B', and C', are collinear.

Solution 1

In this solution, we find the equation of the line through the three image points. We begin by first determining the slope of the line, and then the y-intercept.

$$\text{slope}(A'B') = \frac{8-1}{-5-(-2)} = -\frac{7}{3}$$

Since $A'(-2, 1)$ lies on the line, we can substitute $x = -2$, $y = 1$, and $m = -\frac{7}{3}$ into $y = mx + b$.

$$\begin{aligned} 1 &= -\frac{7}{3}(-2) + b \\ 1 &= \frac{14}{3} + b \\ b &= 1 - \frac{14}{3} \\ &= -\frac{11}{3} \end{aligned}$$

Thus, the equation of the line through the three image points is $y = -\frac{7}{3}x - \frac{11}{3}$. Since the point $C'(1, -c)$ lies on this line, we can substitute $x = 1$ and $y = -c$ into the equation to solve for c.

Thus, $-c = -\frac{7}{3}(1) - \frac{11}{3} = -\frac{18}{3} = -6$ and $c = 6$ follows.

Therefore, the coordinates of C are $(1, 6)$.

Solution 2

Since $A'(-2, 1)$, $B'(-5, 8)$, and $C'(1, -c)$ are collinear, $\text{slope}(A'B') = \text{slope}(B'C')$.

$$\begin{aligned} \text{slope}(A'B') &= \text{slope}(B'C') \\ \frac{8-1}{-5-(-2)} &= \frac{-c-8}{1-(-5)} \\ \frac{7}{-3} &= \frac{-c-8}{6} \\ 42 &= 3c + 24 \\ 18 &= 3c \\ 6 &= c \end{aligned}$$

Therefore, the coordinates of C are $(1, 6)$.