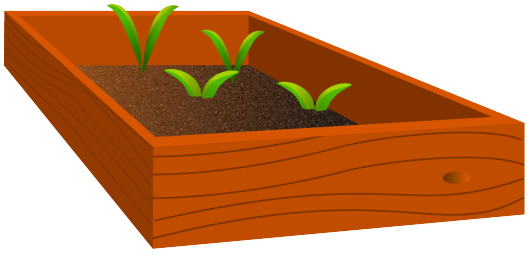
Problem of the Week
Problem B and Solution
Jordyn's Garden

Problem

Jordyn's neighbourhood is building a community garden to grow some vegetables. They would like the garden bed to have an area of 48 square metres, and plan to put wooden fence boards around the edges of the garden bed. To reduce the cost of the project, they would like the garden bed to have the smallest possible perimeter.

(a) Determine the length and width of Jordyn's community garden bed. Assume the side lengths are whole numbers, in metres.

(b) The community decided to double the area of the garden bed, but would still like it to have the smallest possible perimeter. Again, assume the side lengths are whole numbers, in metres. Determine the length and width of the garden bed now.



Solution

(a) Since the garden bed is in the shape of a rectangle and has an area of 48 square metres, it follows that length $\times$ width $= 48$. To determine the length and width, we need to find pairs of whole numbers that multiply to 48. These are called the factor pairs of 48, and are as follows: 1 and 48, 2 and 24, 3 and 16, 4 and 12, and 6 and 8. Since we want the garden bed to have the smallest possible perimeter, we will calculate the perimeter for each pair. These are summarized in the table.

Width (metres)	Length (metres)	Perimeter (metres)
1	48	$2 \times (1 + 48) = 2 \times 49 = 98$
2	24	$2 \times (2 + 24) = 2 \times 26 = 52$
3	16	$2 \times (3 + 16) = 2 \times 19 = 38$
4	12	$2 \times (4 + 12) = 2 \times 16 = 32$
6	8	$2 \times (6 + 8) = 2 \times 14 = 28$

Therefore, in order to have the smallest perimeter, the length of the garden bed should be 8 metres and the width should be 6 metres.

(b) After they double the area of the garden bed it will have an area of $2 \times 48 = 96$ square metres. Using a similar approach to part (a), we need to find the factor pairs of 96, which are: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, and 8 and 12. Since we want the garden bed to have the smallest possible perimeter, we will calculate the perimeter for each pair. These are summarized in the table.

Width (metres)	Length (metres)	Perimeter (metres)
1	96	$2 \times (1 + 96) = 2 \times 97 = 194$
2	48	$2 \times (2 + 48) = 2 \times 50 = 100$
3	32	$2 \times (3 + 32) = 2 \times 35 = 70$
4	24	$2 \times (4 + 24) = 2 \times 28 = 56$
6	16	$2 \times (6 + 16) = 2 \times 22 = 44$
8	12	$2 \times (8 + 12) = 2 \times 20 = 40$

Therefore, in order to have the smallest perimeter, the length of the garden bed should be 12 metres and the width should be 8 metres.

EXTENSION: Note that in each case, the minimum perimeter occurs for the factor pair whose positive difference is the smallest. Will this always happen? Why or why not?